MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 38

Total Marks : 32

Max. Time : 37 min.

Topics : Application of Derivatives, Limits

Type of Questions		**M.M., Min.**
Single choice Objective (no negative marking) Q. 1	**(3 marks, 3 min.)**	**[3, 3]**
Multiple choice objective (no negative marking) Q.2	**(5 marks, 4 min.)**	**[5, 4]**
Subjective Questions (no negative marking) Q. 3,4,5,6,7,8	**(4 marks, 5 min.)**	**[24, 30]**

1. At (0, 0), the curve $y^2 = x^3 + x^2$

(A) touches X-axis
(B) bisects the angle between the axes
(C) makes an angle of 60° with OX
(D) none of these

2. Let $f(\theta) = \dfrac{1+\sin\theta}{5+4\cos\theta}$, then

(A) $\dfrac{1}{5} \le f(\theta) \le 1$
(B) $0 \le f(\theta) \le 3$
(C) in $(0, \pi/2)$, $f(\theta)$ is increasing
(D) none of these

3. Find the number of critical points of the following functions.

(i) $f(x) = -\dfrac{3}{4}x^4 - 8x^3 - \dfrac{45}{2}x^2 + 105$; $x \in R$

(ii) $f(x) = |x-2| + |x+1|$; $x \in R$

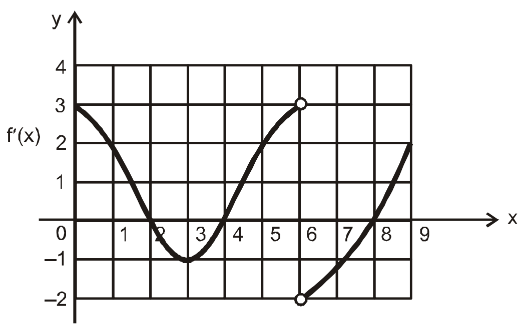
(iii) $f(x) = \min(\tan x, \cot x)$; $x \in (0, \pi)$

4. Discuss monotonocity of the function Q(x), where $Q(x) = 2f\left(\dfrac{x^2}{2}\right) + f(6-x^2)$, $\forall x \in R$ & $f'' > 0$.

5. The number of distinct tangents to the curve $y^2 - 2x^3 - 4y + 8 = 0$ which pass through the point (1, 2) is

6. If $\lim\limits_{x\to 3}\left(\dfrac{\sqrt{2x+3}-x}{\sqrt{x+1}-x+1}\right)^{\frac{x-1-\sqrt{x^2-5}}{x^2-5x+6}}$ can be expressed in the form $\dfrac{a\sqrt{b}}{c}$ where a, b, c, $\in$ N, then find the least value of $(a^2 + b^2 + c^2)$.

7. The graph of the derivative f′ of a continuous function f is shown with f(0) = 0, then for f(x) find



(i) Intervals of monotonicity

(ii) Points of local minima-maxima .

(iii) Intervals of concavity

(iv) Points of inflection

(v) Critical points

8. P(x) is a polynomial function with real coefficients. Let a, b $\in$ R with a < b, are two consecutive roots of the equation P(x) = 0, then show that there exists atleast one 'c' such that $a \le c \le b$ and $P'(c) + 100\,P(c) = 0$.

Answers Key

1. (B) **2.** (B)(C)

3. (i) 3 points, x = 0, –3, – 5
(ii) ∞ points, $x \in [-1, 2]$
(iii) 2 points, $x = \frac{\pi}{4}, \frac{3\pi}{4}$

4. M.I. in $[-2, 0] \cup [2, \infty)$ & M.D. in $(-\infty, -2] \cup [0, 2)$

5. 2 **6.** 29

7. (i) MI $x \in [0, 2] \cup [4, 6) \cup [8, 9]$, MD $[2, 4] \cup (6, 8]$
(ii) Local minima x = 0, 4, 8, Local maxima x = 2, 6, 9
(iii) Concaveup $x \in [3, 6) \cup (6, 9]$,
Concavedown $x \in [0, 3)$
(iv) Inflection point x = 3
(v) Critical points 2, 4, 6, 8